

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Algebra

Solving equations with two or more steps

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

$$2x = 10 + 7 \text{ or } 2x = 17$$

8.5

oe
SC1 1.5

M1

A1

[2]

Q2.

$$\frac{x}{7} = 4 + 3 \text{ or } \frac{x}{7} = 7$$

$$\text{or } x - 3 \times 7 = 7 \times 4$$

$$\text{or } x - 21 = 28$$

$$4 \rightarrow +3 \rightarrow \times 7$$
$$\text{or } (4 + 3) \times 7$$

49

M1

A1

Additional Guidance

$$4 + 3 \times 7 = 25$$

M0A0

[2]

Q3.

Alternative method 1

$$3x = 19 + 8 \text{ or } 3x = 27$$

or

$$(19 + 8) \div 3 \text{ or } \frac{27}{3}$$

accept in 'flow chart'

$$\text{eg } (x \rightarrow) \times 3 \rightarrow -8 \rightarrow 8 \rightarrow 19$$

and $\leftarrow \div 3 \leftarrow +8 \leftarrow 19$ enough for M1

9

M1

A1

Alternative method 2

$$x - \frac{8}{3} = \frac{19}{3}$$

9

M1

A1

Additional Guidance

$$3 \times 9 - 8 (= 19)$$

M1A0

[2]

Q4.

$$6x = 13 + 11 \text{ or } 6x = 24$$

$$\text{or } \frac{24}{6}$$

$$\text{oe eg } -6x = -13 - 11 \text{ or } -6x = -24$$

$$\text{or } \frac{-24}{-6}$$

M1

4

A1

Additional Guidance

Embedded answer, eg $6 \times 4 - 11 = 13$

M1A0

24 with no other working

M0A0

Flow chart method, if 4 not given as the answer

$$x \rightarrow \times 6 \rightarrow -11 \rightarrow 13 \text{ and } 13 \rightarrow +11 \rightarrow \div 6 \rightarrow x$$

M1A0

[2]

Q5.

$$2w = \frac{4}{5} \times 15 \text{ or } 2w = \frac{60}{5}$$

$$\text{or } 2w = 12$$

$$\text{or } \frac{2w}{15} = \frac{12}{15}$$

$$\text{or } \frac{w}{5} = \frac{2}{1} \text{ or } \frac{w}{2} = \frac{3}{1}$$

$$\text{or } \frac{w}{15} = \frac{4}{5} \div 2 \text{ or } \frac{w}{15} = \frac{2}{5}$$

$$\text{or } 2w \times 5 = 4 \times 15 \text{ or } 10w = 60$$

$$\text{or } \frac{4}{5} \div \frac{2}{15}$$

oe in the form $aw = n$ where a is an integer and n is an integer, fraction or decimal

oe in the form $\frac{bw}{x} = \frac{c}{x}$ where x is a common denominator

oe calculation

M1

6

A1

Additional Guidance

Embedded answer 6 eg

$$\frac{2 \times 6}{15} = \frac{4}{5}$$

M1A0

[2]

Q6.

$$\frac{3}{1000}$$

B1

[1]

Q7.

$$2x = 27 - 3 \text{ or } 2x = 24$$

or

$$\frac{27-3}{2} \text{ or } \frac{24}{2}$$

or

$$x + 1.5 = 13.5$$

$$\text{oe eg } 2x + 3 = 27$$

$$-3 \quad -3$$

$$\text{or } -2x = 3 - 27 \text{ or } -2x = -24$$

M1

12

A1

Additional Guidance

Embedded answer with no or incorrect answer

eg $2 \times 12 + 3 = 27$ without 12 selected or with incorrect answer

M1A0

Trial and improvement with answer 12

M1A1

Trial and improvement with no answer or answer other than 12

M0A0

[2]

Q8.

$$10x = 21 + 3 \text{ or } 10x = 24$$

or

$$(21 + 3) \div 10 \text{ or } 24 \div 10$$

$$\text{oe eg } -10x = -3 - 21$$

M1

2.4

$$\text{oe eg } \frac{24}{10} \text{ or } \frac{12}{5} \text{ or } 2\frac{4}{10} \text{ or } 2\frac{2}{5}$$

$$\text{SC1 } 1.8 \text{ oe}$$

A1

Additional Guidance

$$10x - 3 + 3 = 21 + 3$$

M1

$$10x - 3 = 21 + 3 \text{ or } 10x - 3 + 3 = 21 \text{ unless recovered}$$

M0

$$10x \div 10 - 3 \div 10 = 21 \div 10$$

M1

$$10x \div 10 - 3 = 21 \div 10 \text{ unless recovered}$$

M0

Embedded answer eg $10 \times 2.4 - 3 = 21$ with no or incorrect answer

M1A0

[2]

Q9.

$$4x = 14 + 3 \text{ or } 4x = 17$$

$$\text{or } (14 + 3) \div 4 \text{ or } 17 \div 4$$

$$\text{or } x - \frac{3}{4} = \frac{14}{4}$$

M1

$$4.25 \text{ or } \frac{17}{4} \text{ or } 4\frac{1}{4}$$

A1

Additional Guidance

Embedded answer of 4.25 with 4.25 not selected on answer line
e.g. $4 \times 4.25 - 3 = 14$ with no answer given or answer of 14 or 17

M1A0

14 + 3 and answer 4.25

M1A1

14 + 3 only

M0A0

Trial and improvement with answer 4.25

M1A1

Trial and improvement with no answer or answer other than 4.25

M0A0

$$4.25 \text{ or } \frac{17}{4} \text{ or } 4\frac{1}{4} \text{ seen and then answer 4 given}$$

M1A1

Answer of $\times 4.25$

M1A0

$17 \div 4$ (and no further)

M1A0

[2]

Q10.

$$4x = 39 - 1 \text{ or } 4x = 38$$

or

$$x + \frac{1}{4} = \frac{39}{4}$$

oe

M1

9.5

oe

A1

[2]

Q11.

Alternative method 1: Expands bracket first

$$8d - 20 \quad \text{oe expression eg } 8 \times d - 20$$

M1

$$\text{their } 8d = 28 + \text{their } 20 \quad \text{or} \quad 8d = 48$$

their 20 cannot be zero

M1

6

ft M0M1 or M1M0

A1ft

Alternative method 2: Divides by 4 first

$$2d - 5 = \frac{28}{4} \text{ or } 2d - 5 = 7$$

M1

$$2d = \text{their } 7 + 5 \quad \text{or} \quad 2d = 12$$

their 7 cannot be 28

M1

6

ft M0M1 or M1M0

A1ft

Additional Guidance

Alt 1

$$6d - 5 = 28$$

M0

$$6d = 33$$

M1

$$d = \frac{33}{6}$$

A1ft

Alt 2

$$2d - 5 = 24$$

M0

$$2d = 29$$

M1

$$d = 14.5$$

A1ft

Answer 6 from trial

M1M1A1

$$\text{Embedded answer } 4(2 \times 6 - 5) = 28$$

M1M1A0

ft answers correct to 1 dp or better

[3]

Q12.

Alternative method 1 – multiplies through by 10 or common denominator of 10

$$5(x + 8) + 2(9 - x)$$

or

$$5x + 40 + 18 - 2x$$

oe

numerator on the left-hand side if written as a fraction

allow one error or omission in the expansion if brackets not seen

$$\text{eg } 5x + 18 - 2x$$

M1

$$3x + 58$$

may be implied by eg $3x + 18 = 0$ or $3x = -18$

A1

$$\text{their } (3x + 58) = 4 \times (\text{their } 10)$$

$$\text{or their } (3x + 58) = 40$$

$$\text{or } 3x + 18 = 0$$

$$\text{or } 3x = -18$$

oe

allow an unsimplified expression for their $(3x + 58)$

equation may be implied by answer

M1

$$-6$$

ft M1A0M1

A1ft

Alternative method 2 – collects terms with fractions

$$\frac{x}{2} + 4 + \frac{9}{5} - \frac{x}{5}$$

oe eg $0.5x + 4 + 1.8 - 0.2x$

allow one error

M1

$$\frac{3}{10}x + \frac{29}{5}$$

oe eg $0.3x + 5.8$

A1

$$\frac{3}{10}x = \frac{20}{5} - \frac{29}{5}$$

$$\text{or } \frac{3}{10}x = -\frac{9}{5}$$

oe eg $0.3x = -1.8$

terms must be collected

M1

$$-6$$

ft M1A0M1

A1ft

Additional Guidance

Accept decimal answers for follow through correct to 1 dp or better

Apply the principles of alt 1 for any use of other common denominators

eg common denominator of 20 (or multiplication through by 20)

$$10(x + 8) + 4(9 - x) = 6x + 116$$

M1A1

$$6x + 116 = 80 \quad x = -6$$

M1A1

An incorrect simplification of $5x + 40 + 18 - 2x$ may still gain the third and fourth marks

eg $5x + 40 + 18 - 2x = 3x + 68$ followed by $3x + 68 = 40$ and $x = -\frac{28}{3}$

**M1A0M1
A1ft**

eg $5x + 40 + 18 - 2x = 2x + 68$ followed by $2x + 68 = 40$ and $x = -14$

**M1A0M1
A1ft**

An incorrect denominator may still gain the third and fourth marks:

$$\frac{5x + 40 + 18 - 2x}{7}$$

followed by $5x + 40 + 18 - 2x = 28$ and $x = -10$

**M1A0M1
A1ft**

Denominator not processed

$3x + 58 = 4$ followed by $3x = -54$ and $x = -18$

M1A1M0A0

$$(x + 8) + (9 - x) = 40$$

M0A0M1A0

Two errors in the expansion but with brackets seen may go on to get the third and fourth marks

$$5(x + 8) + 2(9 - x) = 5x + 8 + 18 - x$$

1st M1A0

Two errors in the expansion and no brackets seen, no follow through allowed

$$5x + 8 + 18 - x \text{ followed by } 4x + 26 = 40 \text{ and } x = \frac{14}{4}$$

M0A0M1A0

[4]